Sample Proportion Sampling Dist: $\mu_{\hat{p}} = p$ and $\sigma_p = \sqrt{\frac{p(1-p)}{n}}$

Sample Proportion Sampling Dist z-score: $Z = \frac{\hat{p}-\mu_p}{\sigma_p} = \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$

Sample Mean Sampling Distribution: $\mu_x = \mu_{\bar{x}}$ and $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$

Sample Mean Sampling Dist z-score: $Z = \frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x}-\mu_x}{\frac{\sigma_x}{\sqrt{n}}}$

1) The mean of all possible sample means of size n equals the __mean__ of the population.

2) The standard deviation of all possible sample means of size n equals the population standard deviation divided by __n__. (What goes in the brackets below?)

$$\sigma_{\bar{x}} = \frac{\sigma_x}{[n]}$$

3) The mean of all possible sample proportions of size n equals the population __proportion__.

4) The standard deviation of all possible sample proportions of size n equals the population proportion times one minus the population proportion divided __by n__. (What goes in the brackets below?)

$$\sigma_{\hat{p}} = \sqrt{\frac{p*(1-p)}{[n]}}$$

5) The central limit theorem tells us that the distribution of the sample mean approximately follows which distribution when the sample size is large?

The normal distribution.

6) The population mean annual salary for a Red Sox player is \$6 million dollars with a standard deviation of \$1.5 million dollars.

$$\mu_x = 6{,}000{,}000$$
$$\sigma_x = 1{,}500{,}000$$

a. What is the mean of the sampling distribution of the sample mean salary for $n = 35$ Red Sox players?

$$\mu_{\bar{x}} = \mu_x = 6{,}000{,}000$$

b. What is the standard error of the sampling distribution of the sample mean where $n = 35$?

$$\sigma_{\bar{x}} = \sigma_x/\sqrt{n} = \frac{1{,}500{,}000}{\sqrt{35}} = 253546.2764$$

c. Find the probability that the sample mean salary for $n = 35$ Red Sox Players is less than \$5 million. Give answer to 4 decimal places and write out a sentence about what it means.

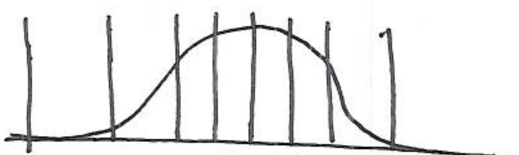
$$P(\bar{X} < 5{,}000{,}000) = P\left(Z < \frac{5{,}000{,}000 - 6{,}000{,}000}{253{,}546.2764}\right)$$

we use $\sigma_{\bar{x}}$ because our observation is $\bar{x}$

$$\Rightarrow Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu_x}{\sigma_x/\sqrt{n}}$$

$$= P(Z < -3.94405)$$
$$\approx 0$$

↑ The probability the sample mean of 35 players is less than \$5 million is about zero.



$\boxed{-3.94405}$

↑ Not to scale :(

d. Find the probability that the sample mean salary for $n = 3$ Red Sox Players is less than \$5 million. Give answer to 4 decimal places and write out a sentence about what it means. It is okay to assume salaries are normally distributed to answer this question.

$$\sigma_{\bar{x}} = \sigma_x/\sqrt{n} = \frac{1{,}500{,}000}{\sqrt{3}} = 866{,}025.4038$$

$$P(\bar{x} < 5{,}000{,}000) = P\left(Z < \frac{5{,}000{,}000 - 6{,}000{,}000}{866{,}025.4038}\right)$$
$$= P(Z < -1.1547)$$
$$\approx P(Z < -1.15)$$
$$= .1251$$

↑ The probability the sample mean of 3 players is less than \$5 million is about .1251. We note that this is more likely than the sample mean of 35 players.

7) The population proportion of home games won by the Red Sox is .56.

$p = .56$

a. What is the mean of the sampling distribution of the sample proportion of games won by the Red Sox for $n = 81$ games?

$\mu_{\hat{p}} = p = .56$

b. What is the standard error of the sampling distribution of the sample proportion where $n = 81$?

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.56(1-.56)}{81}} = \sqrt{.0030419753} = .0552$$

c. Find the probability that the sample proportion of games won for $n = 81$ games is less than .5. Give answer to 4 decimal places and write out a sentence about what it means.

$$P(\hat{p} < .5) = P\left(z < \frac{.5-.56}{.0552}\right) = P(Z < -1.08696)$$

we use $\sigma_{\hat{p}}$ because our observation is a proportion.

$$\approx P(Z < -1.09)$$
$$= .1379$$

The probability that the sample proportion of games won for $n = 81$ is less than .5 is .1379. The probability the Red Sox lost more than half of the games in a random sample of 81 is .1379.

d. Find the probability that the sample proportion of games won for $n = 40$ games is less than .5. Give answer to 4 decimal places and write out a sentence about what it means.

$$\sigma_{\hat{p}} = \sqrt{\frac{.56(1-.56)}{40}} = \sqrt{.00616} = .0785$$

$$P(\hat{p} < .5) = P\left(z < \frac{.5-.56}{.0785}\right) = P(z < .7643312)$$
$$\approx P(z < .76)$$
$$= .7764$$

The probability that the sample proportion of games won for $n = 40$ is lessthan .5 is .7764. The probability the Red Sox lost more than half of the games in a random sample of 40 is .7764. We note that this is more likely than a sample proportion of $n = 81$

8) The population mean self-attractiveness rating for our class was 6.95 with a standard deviation of 1.72. It is safe to assume self-attractiveness ratings are normally distributed. Complete parts a through c.

$$\mu_x = 6.95$$
$$\sigma_x = 1.72$$

a. What is the mean of the sampling distribution of the sample mean self-attractiveness rating for n = 3 students?

$$\mu_{\bar{x}} = \mu_x = 6.95$$

b. What is the standard error of the sampling distribution of the sample mean self-attractiveness rating where n= 3 students?

$$\sigma_{\bar{x}} = \sigma_x/\sqrt{n} = 1.72/\sqrt{3} = .99304$$

c. Find the probability that the sample mean self-attractiveness rating for n = 3 students is less than 5. Give answer to 4 decimal places and write out a sentence about what it means.

$$P(\bar{X} < 5) = P\left(Z < \frac{5-6.95}{.99304}\right) = P(Z < -1.96366)$$

Standard error of $\bar{x}$ $\approx P(Z < -1.96) = .0250$

The probability that the sample mean rating for $n=3$ students is less than 5 is .0250. It is unlikely that we see a sample mean for $n=3$ with a rating less than 5

d. Find the probability that a randomly selected person has a self-attractiveness rating less than 5. Give answer to 4 decimal places and write out a sentence about what it means.

$$P(X < 5) = P\left(Z < \frac{5-6.95}{1.72}\right) = P(Z < -1.13372)$$

Standard deviation of X $\approx P(Z < -1.13) = .1292$

The probability that a randomly selected person has a self-attractiveness rating less than 5 is .1291. We note this is much higher than .0250 in part c.

9) The population proportion of students that rated themselves higher than the rest of the class is 0.61. Complete parts a and b.

$p = .61$

a. What is the mean of the sampling distribution of the sample proportion of students that rated themselves higher than the rest of the class for $n = 4$ students?

$\mu_{\hat{p}} = p = .61$

b. What is the standard error of the sampling distribution of the sample proportion of students that rated themselves higher than the rest of the class for $n = 4$ students?

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.61(1-.61)}{4}} = \sqrt{.059475} = .243874968$

c. Find the probability that most of the sample proportion of students rated themselves higher than the rest of the class for n = 4 students. Give answer to 4 decimal places and write out a sentence about what it means.

$P(\hat{p} > .5) = P\left(z > \frac{.5 - .61}{.243874968}\right) = P(z > -.45105)$

$\approx 1 - P(z < -.45) = 1 - .3264 = .6736$

↑ The probability that most of the sample proportion, of $n=4$ students, [redacted] rated themselves higher than the rest of the class is .6736.

d. Find the probability that most of the sample proportion of students rated themselves higher than the rest of the class for n = 8 students. Give answer to 4 decimal places and write out a sentence about what it means.

$P(\hat{p} > .5) = P\left(z > \frac{.5 - .61}{\sqrt{\frac{.61(1-.61)}{8}}}\right) = P\left(z > \frac{.5 - .61}{.1724456}\right)$

$= P(z > -.6378822)$

$\approx 1 - P(z < -.64) = 1 - .2611 = .7389$

↑ The probability that most of the sample [redacted] proportion, of $n=8$ students, rated themselves higher than the rest of the class is .7389 [redacted] which is higher than in part c.